Your Brain on Math:

Making Sense of Number Sense

Sara Stetson

NHTM Spring Conference

March 17, 2014

Agenda

- **What is number sense?**
- **Core Systems of number:**

-Core system 1: Approximate Number System

-Core System 2: Exact Number Object Attention System

- **Space and number: Integer representation**
- **Your Brain on Math**

- Dyscalculia

- **Recommendations for instruction**

Billion	# Students	Days of Counting	
	1	.20 days	
	2	3 days	
	7	7 days	
	1	14 days	
	1	15 days	
	1	17.5 days	
	1	31 days	
	1	4380 days	12 Years
	1	6935 days	19 Years
	1	7300 days	20 Years
	1	10000 days	27.4 Years
	1	694449 days	1902.6 Years

What is Number Sense?

- Infants
- Animals
- Indigenous People
- Brain studies




The Approximate Number System

Approximate Number System

- Fight or Flight

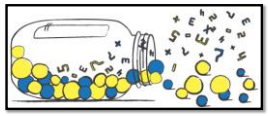
- Foraging

vs.

Approximate Number System

Number Sense

- Number sense and the Approximate Number System are the same thing



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Approximate Number System

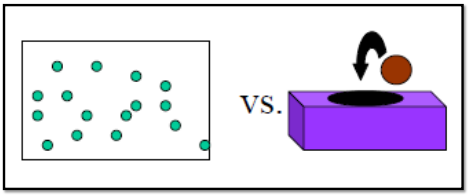
- Modality invariance

- Objects, sounds, actions

(e.g., Brannon, 2001; Dehaene, et al., 2004; Lipton & Spelke, 2003; McCrink & Wynn, 2005; Wood & Spelke, 2005; Xu, 2002)

Approximate Number System

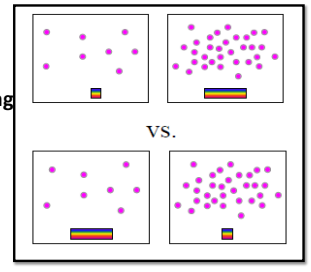
- No tracking of individual elements
- Gross estimation: Quantity comparison, addition, and subtraction



(Brannon, 2001; Xu, 2002; Lipton & Spelke, 2003; Wood & Spelke, 2005; McCrink & Wynn, 2005; deHevia& Spelke, 2008)

Approximate Number System

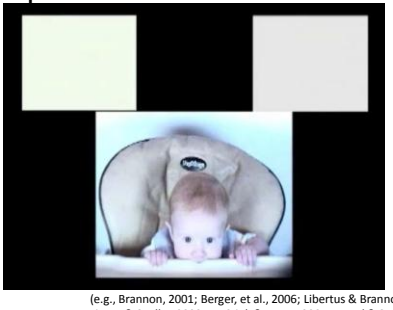
- Spontaneous linkage to space
- A natural propensity to represent quantity along a mental number line



(deHevia& Spelke, 2008)

Approximate Number System

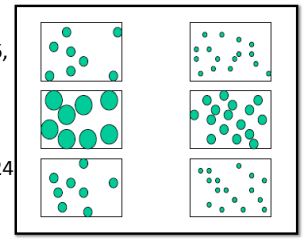
- Ratio-dependence



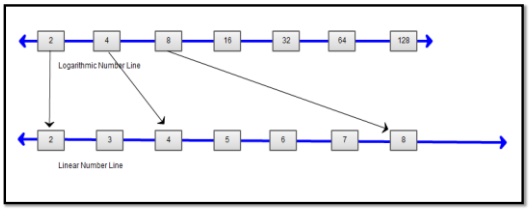
(e.g., Brannon, 2001; Berger, et al., 2006; Libertus & Brannon, 2009 Lipton & Spelke, 2003; McCrink & Wynn, 2005; Wood & Spelke, 2005)

Approximate Number System: Ratio Dependence

- 6 month-old babies
  - Discriminate 8 vs. 16,
  - But not 8 vs. 12,
  - Ratio 1:2
- 9 month-old babies
  - Discriminate 16 vs. 24
  - Ratio 2:3
- Adults: ratio 9:10



What is Ratio-Dependence?

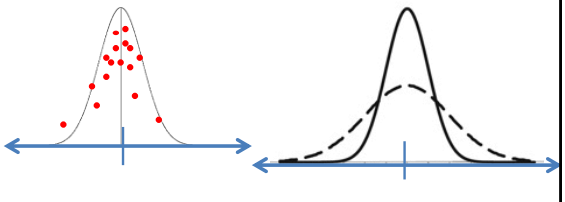


(Dehaene, 2003; Gallistel & Gelman, 2000; Huntley-Fenner, 2001; Temple & Posner, 1998)

Approximate Number System Precision

Representation of a number

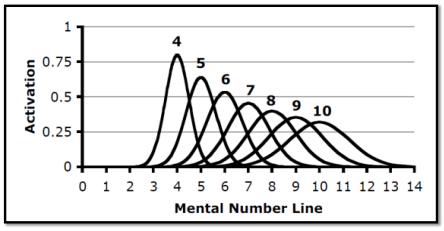
Low variability vs. High variability



Target Number

(Neider, 2004; Neider & Miller, 2003)

Approximate Number System Precision



(Halberda, in press)

Weber's Law

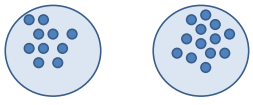
- If the intensity of a sensation is doubled from an initial value and the sensation is noted, the intensity will have to be doubled again to give the same perception of increase in sensation

X 10

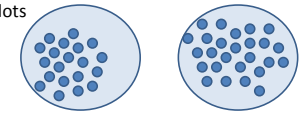
X 100

Weber's Law and Number

- Discriminate 13 from 10 at 90%
- Numerical distance of 3



- Double the reference array to 20:
- How far from this numerosity do we need to go to again reach 90% correct discrimination?
- We need to double the numerical distance to 6, presenting an array of 26 dots

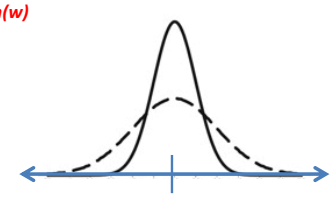


Approximate Number System Precision

Low Weber fraction= low variability and more precision

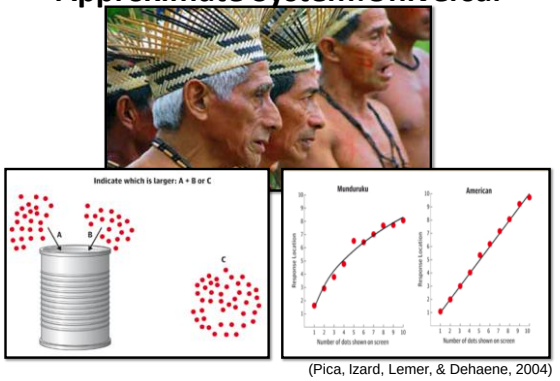
High Weber fraction = high variability and less precision

SD = *n(w)*



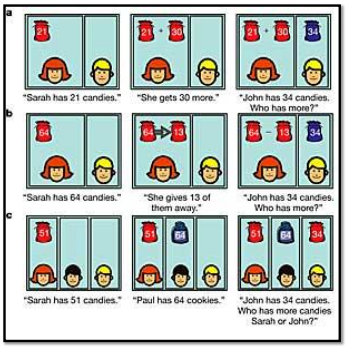
(Halberda, in press; Neider, 2004; Neider & Miller, 2003)

Approximate System:Universal



(Pica, Izard, Lemer, & Dehaene, 2004)

ANS and Math



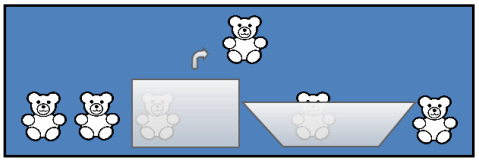
Gilmore et al. (2007)

The Exact Number System

The Exact Number System

Infant Addition and Subtraction

- Infant object permanence
- Infant number processing

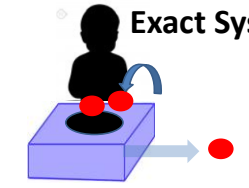


(Wynn, 1992)

Infant Addition and Subtraction

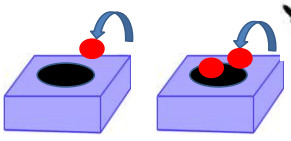
Berger, et al 2006

Exact System: Limits



- Set Size Limit: 3-4
- Explicit number or object files
- Numerical cognition or object-based attention?

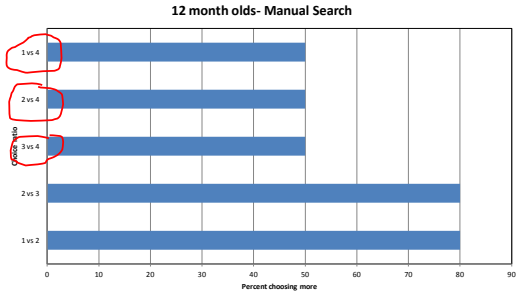
Persistence Task



Search Task

(Feigenson& Carey, 2003; Feigenson, Carey & Hauser, 2002; VanMarle, 2005)

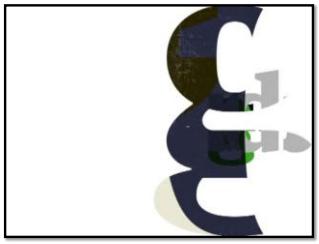
Exact System: Limits



(Adapted from Hauser, et al., 2000)

Exact System: Universal

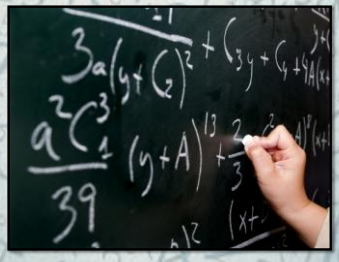
- Set Size Limit: 3-4
- Explicit number or object files
- Domain limit– objects
- Numerical cognition or object-based attention?



(Rugani, Regolin, & Vallortigara, 2010)

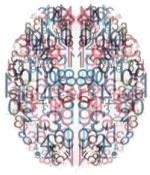
The Linear Number Properties

The log-to-linear shift is hard



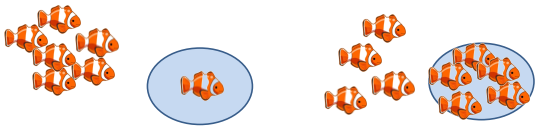
Why is Number Mapping Difficult?

- Requires a log-to-linear shift
- There are several number properties
- Most number properties are spatial but numbers are expressed symbolically
- Neither core system represents integers



Cardinality Principle

- Put one fish in the pond
- Put two fish in the pond

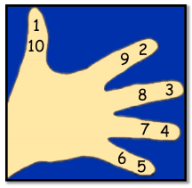


2 yrs. _____ 5 yrs.

(Butterworth, 2005; Carey & Sarnecka, 2006, 2008; Lipton & Spelke, 2006; Wynn, 1990)

1:1 Principle

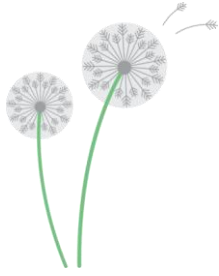
- **Count each item only once**



(Butterworth, 2006; Gelman & Gallistel, 1978; Wynn, 1990)

Abstraction Principle

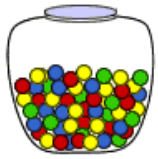
- **Wishes and dreams**



(Butterworth, 2006; Gelman & Gallistel, 1978; Wynn, 1990)

Order Irrelevance Principle

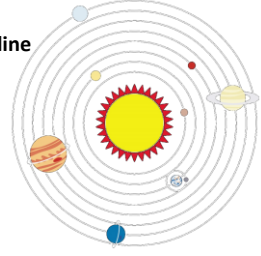
- Start anywhere



(Butterworth, 2006; Gelman & Gallistel, 1978; Wynn, 1990)

Stable Order Principle

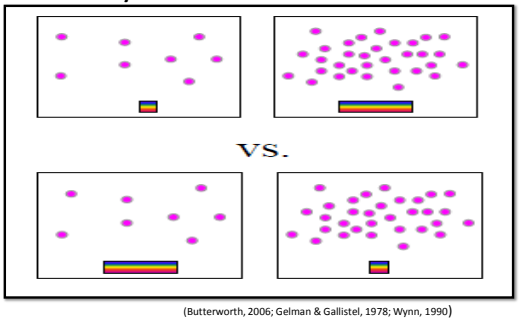
- Start at the beginning
- Go in order
- Mental Numberline



(Butterworth, 2006; Gelman & Gallistel, 1978; Wynn, 1990)

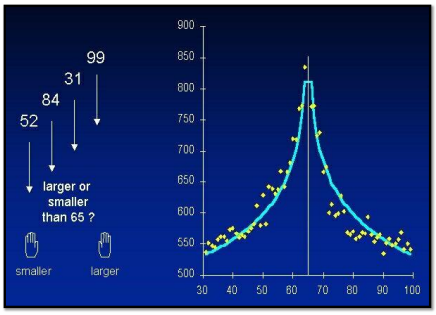
Successor Function

- How many vs. how much

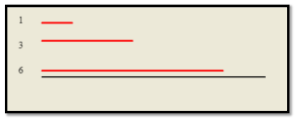


(Butterworth, 2006; Gelman & Gallistel, 1978; Wynn, 1990)

Number and Space



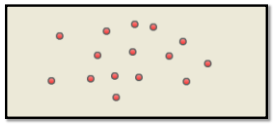
Numbers and Space: Continuous Extent



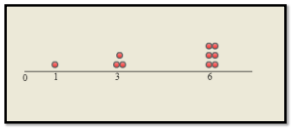
(Brannon, Lutz, & Cordes, 2006; Cordes & Brannon, 2009; De Hevia & Spelke, 2010)

ANS and Space: Number vs. Continuous Extent

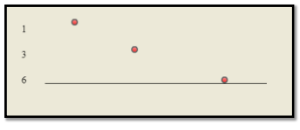
Numbers and Space: Magnitude



Numbers and Space: Relative Magnitude



Numbers and Space: Position

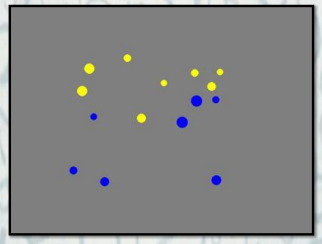


(Cantlon & Brannon, 2006;Lyons & Beilock, 2011; Picozi, et al., 2010)

Integer Representation

• **Approximate System**		• **Exact System**
- Approximate	→	- Exact
- Sets	→	- Individuals
- Continuous extent	→	- Discrete objects
- No limit	→	- Limit 3-4
- Multi-modal	→	- Visual
- Spatial = Mental numberline	→	- Object-based = No numberline
- No Lanugage	→	- Language (eg., 1, 2, 3)

Measuring Number Sense

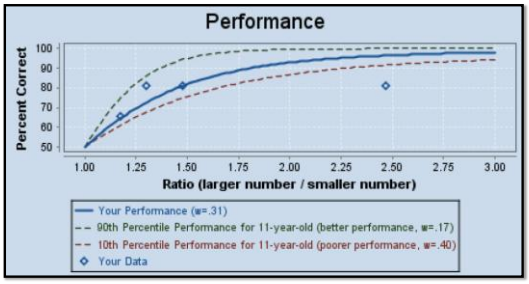


How do We Measure Number Sense?

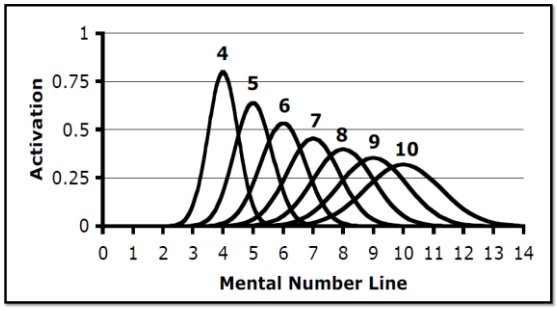
- Quantity Comparison

(Halberda et al., 2008)

Quantity Comparison: Sample Output

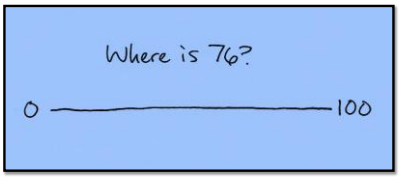


Quantity Comparison: Sample Output



How do We Measure Number Sense?

- Number-to-Position



(Halberda et al., 2008; Siegler & Booth, 2004)

Number-to-Position: Sample Output

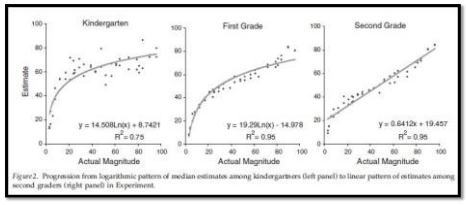
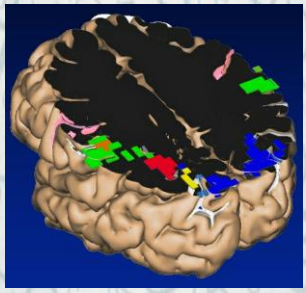


Figure2. Progression from logarithmic pattern of median estimates among kindergartners (left panel) to linear pattern of estimates among second graders (right panel) in Experiment.

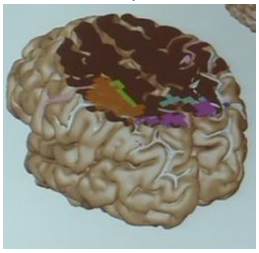
(Siegler & Booth, 2004)

Dyscalculia



Your Brain on Math

- Horizontal Intraparietal Sulcus



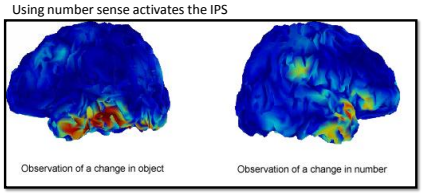
- Calculation
- Calculation/Language
- Grasping objects
- Manual tasks
- Visuo-spatial tasks
- Attention
- Saccades (rapid eye fixations)

(Adapted from Dehaene, 2013)

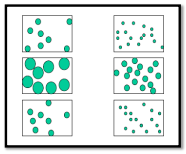
(Simon, Mangin, Le Bihan, Cohen, & Dehaene, 2002)

Your Brain on Math

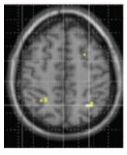
Using number sense activates the IPS



Tasks that activate this region:
- Comparison of numbers
- Subtraction
- Approximation
- Thinking about space
- Non-Symbolic Tasks

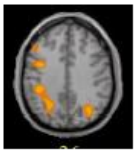


Brains on Math



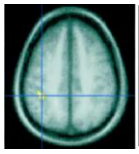
Neurotypical brain processes numerosities in Intraparietal Sulcus

Castelli et al, 2006



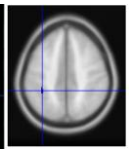
Part of the calculation network

Zago et al, 2001



Is structurally abnormal in dyscalculics

Isaacs et al, 2001



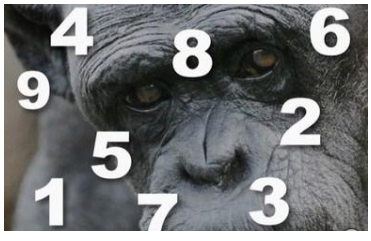
Twin study shows
- Neural and behavioural abnormalities correlate
- Both are heritable

Ranpura et al., 2013

(slide adapted from Butterworth, 2013)

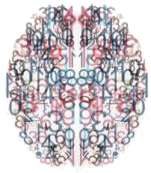
Brains on Math

- Why is numerical and spatial information related/confused?
- Monkeys taught to discriminate number and length (continuous extent)
- Neurons that encode number, length/space, and both



Dyscalculia

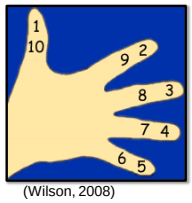
- Severe difficulty in mathematics, not explained by general cognitive difficulties or educational opportunities
- Prevalence: around 5-6% (same as dyslexia!)
- Fraction of the studies compared to dyslexia



(Butterworth, 2002; Shalev & Gross-Tsur, 2001; Geary, 1993, 2004)

Observable Deficits

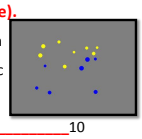
- Counting strategies (eg., counting on from larger vs. counting all)
- Prolonged use of finger counting (slow, inaccurate, finger agnosia)
- Decomposing numbers (e.g. recognizing that 10 is made up of 5 and 5)
- Place value
- Multi-step procedures



(Wilson, 2008)

Cognitive Deficits

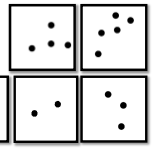
- **Difficulty representing quantity (number sense).**
  - Large Weber fraction/poor quantity comparison
  - Mental Numberline slow to develop/logarithmic



Where is 7?

0_____10

  - Slow to enumerate set of objects



  - Slow to subitize sets of 1-4 objects
    Number symbols not automatic

  - Number stroop task
    (Rouselle & Nöel, 2007)

Subtypes

- Number sense

=

- Spatial

- Verbal
  - Difficulty with counting, fact retrieval, word problems
  - Associated with dyslexia?
- Executive
  - Difficulty with fact retrieval, use of strategy/procedure
  - Associated with ADHD?

Wilson & Dehaene (2007)

Instruction

Weber Fraction and Math Performance

- Halberda et al., 2008

➢ 64 teenagers of 14 y.o.

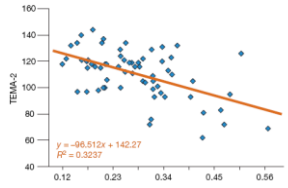
➢ Assessment of the acuity of their number sense

➢ Record of their past school performance in math

- Quantity Comparison

Halberda, Mazzocco & Feigenson (2008)

Weber Fraction and Math Performance



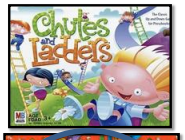
Score at the math test at 8 y.o. (high = good)
W: acuity of number sense, at 14 y.o., low = good

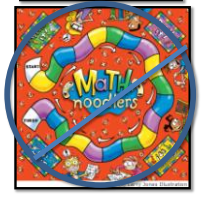
Manipulatives: Continuous Magnitude

- A display of continuous magnitudes involved in trained addition problems promotes log-to-linear mapping
- Babies can map approximate sets to length (Spelke) and so this tendency is strong

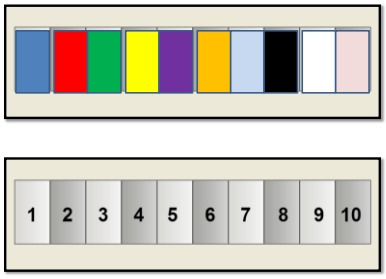
Board Games: Continuous Magnitude

- Linear but not circular board game play encourages log-to-linear shifting
- Explicit feedback on number-line anchoring (eg., calibration priming) encourages log-to-linear mapping





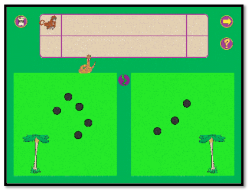
Color/Number

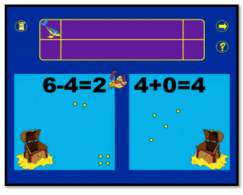


(Ramani & Siegler, 2008

Adaptive CAI

- Linking digits, sets and space
- Practicing comparison, addition and subtraction

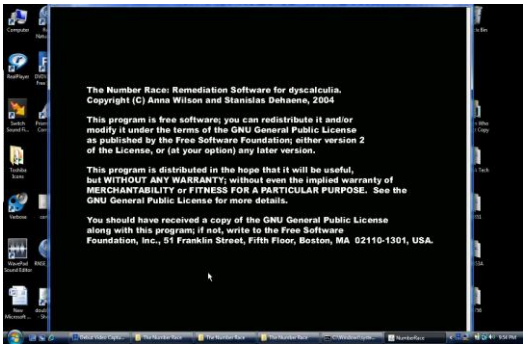


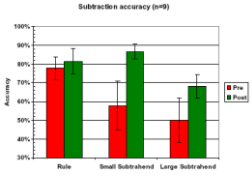


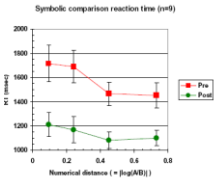
(Adapted from Wilson, 2008)

(Wilson et al., 2006)

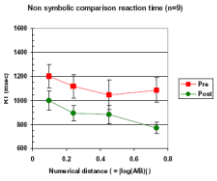
Number Race





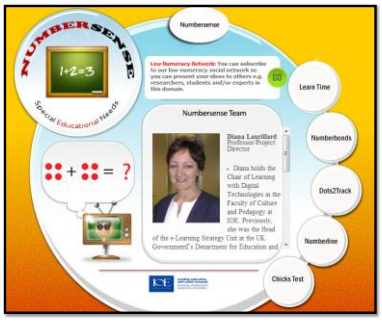


- Faster in dots enumeration
- Increased accuracy in subtraction (but not addition)
- Increased speed in magnitude comparison, but accuracy difference non-significant...



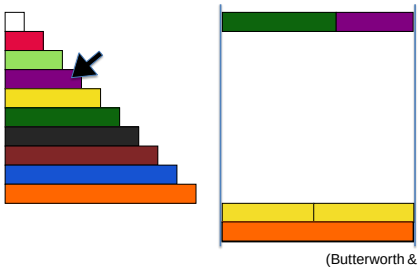
Number Sense Games

- http://number-sense.co.uk/index2.html



Number Bonds

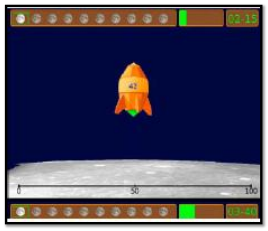
Initially 3 secs to click on matching bond – adjusts to performance
Stages: length + colour, length + colour + digit, length + digit, digit



(Butterworth & Laurillard)

Dyscalculia: Rescue Calcularis

- 16 control and 16 MLD
- 25 sessions, 15 minutes
- 5 weeks
- Improvement in calculation– both groups



(Kucian, et al., 2011)

There's an App for That

Numberline Tap

My Numberline

Numberline Frog

Thank You

- sstetson@rivier.edu
- www.rivierstudy.wordpress.com/