



STANDARDS FOR MATHEMATICAL PRACTICE

Exploring *Putting the Practices Into Action*

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Goal for this session



Provide ideas for exploring the Standards for Mathematical Practices (SMP) across the grades K-8

...to help students to reason, apply and truly come to understand mathematics.

- SMPs 1-8
- Resources
- Understanding the Process
- Suggested teaching strategies
- Assessment Tips
- Additional ideas for implementing the Practices

Putting the Practices Into Action.

Implementing the Common Core Standards for Mathematical Practice K-8.

Susan O'Connell, John SanGiovanni.
Heinemann, Portsmouth NH (2013)



Evolution of SMPs

- Standards based approach to teaching
- Working towards developing *mathematically proficient students*
- *NCTM 1989* Curriculum and Evaluation Standards for School Mathematics
 - Mathematical Processes – Mathematics as
 - Problem Solving
 - Communication
 - Reasoning
 - Connections

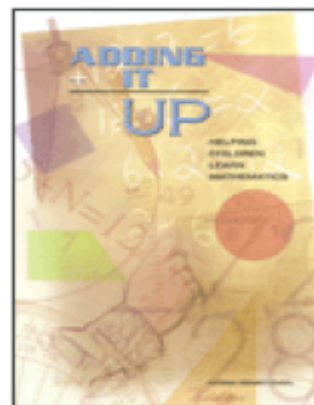
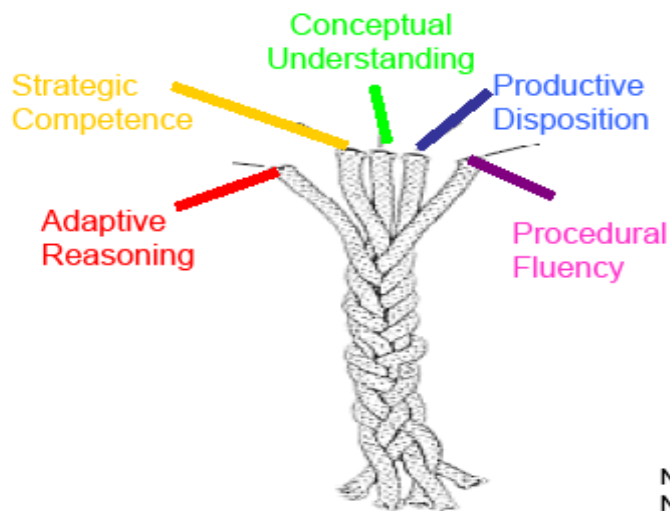


Refining the Standards

- 2000 – NCTM *Principles and Standards for School Mathematics* (blend of content and process)
- *Process standards:*
 - *Problem Solving*
 - *Communications*
 - *Representations*
 - *Reasoning and Proof*
 - *Connections*

Underlying Frameworks

Strands of Mathematical Proficiency



NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.

A recording of today's webinar will be available at:

<http://www.carnegielearning.com/webinars>

<http://ncsmonline.org/events/webinars.html>

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Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – ***comprehension of*** mathematical concepts, operations, and relations
- ***Procedural Fluency*** – ***skill in carrying out*** procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ***ability to formulate,*** represent, and solve mathematical problems
- ***Adaptive Reasoning*** – ***capacity for logical*** thought, reflection, explanation, and justification
- ***Productive Disposition*** – ***habitual inclination to*** see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Again emphasis on the development of thinking, understanding, and application.



CCSS Standards for Mathematical Practice	Correlation to NCTM Process Standards	Strands of Mathematical Proficiency
Make sense of problems and persevere in problem solving	Problem Solving	Strategic competence
Reason abstractly and quantitatively	Representation Communication Reasoning as Proof Problem Solving	Adaptive Reasoning
Construct viable arguments and critique the reasoning of others	Reasoning and Proof Communication Representation	Conceptual understanding Adaptive reasoning
Model with Mathematics	Representation Communication	Strategic competence Conceptual understanding
Use appropriate tools strategically	Problem Solving Reasoning and Proof	Conceptual understanding Procedural fluency
Attend to precision	Communication Representation	Procedural fluency Conceptual understanding
Look for and make use of structure	Reasoning and Proof Problem Solving Representation	Adaptive reasoning Productive disposition
Look for and express regularity and repeated reasoning	Reasoning and Proof Representation Communication	Adaptive reasoning Conceptual understanding Productive disposition

Standards for Mathematical Practice (SMP) - 2010 CCSS



1. Make sense of problems and persevere in solving them

6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure

8. Look for and express regularity in repeated reasoning

Bill McCallum's
blog!

Reasoning and explaining

Modeling and using tools

Seeing structure and generalizing

Overarching habits of mind of a productive mathematical thinker



EXPLORING THE STANDARDS *PUTTING THE PRACTICES INTO ACTION*



Why Problem Solving?

In examining the first Standard for Mathematical Practice we may ask why problem-solving? Computation has been the math focus in the past. We had a page of exercises with one or two word problems at the bottom of the page and we were told they were either right or wrong. Now the ability to solve problems is our 1st Standard. We now know that being able to compute alone does not equate to math proficiency.



Understanding the Standard

Our new definition of proficiency includes *knowing when, why, and how to apply calculations in situations.*

SMP 1 focuses on the development of essential skills and dispositions for becoming effective problem solver including:

1. An understanding of the problem-solving process and how to navigate through the process from start to finish
2. A repertoire of strategies for solving problems and the ability to select a strategy that makes sense for a given problem.
3. The disposition to deal with confusion and perseverance.



SMP 1 - Make Sense of Problems and Persevere in Solving Them

- Understanding the Problem-Solving Process ([Which we identify, discuss, and move toward making this process automatic.](#))
- Developing Strategies
- Building a Problem-Solving Disposition

How do we get there?

- Classroom Techniques
 - Focus on the question
 - Questions to guide student thinking
 - Posing open-ended questions
 - Posing rich math problems
 - Refining guiding questions



Understanding the Problem-Solving Process

Problem solving is not an algorithm to be practiced or a fact to be memorized.

For each *unique* problem – problem solvers decide

- what is being asked,
- what is important to consider,
- an appropriate path to the solution, and
- the reasonableness of their actions.

Problem solvers also think about their own thinking (metacognition) so they are better to regulate and modify their thinking.

From George Polya (1957) – the following questions highlight important steps in the thinking process

- *What is the problem asking?*
- *How should I begin?*
- *Where is the necessary data/*
- *What should I do with that data?*
- *Did my plan work?*
- *Does my answer make sense?*
- *Do I need to go back and try a different strategy?*



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Building a Problem Solving Disposition

Many students become easily frustrated when solving math problems.

Am I able to do this?

What if I get stuck?

What if it takes me too long to get the answer?

What if my idea doesn't work?

What if my answer is wrong?



Disposition

Believing it is possible to solve a problem, recognizing that confusion is part of the process, and discovering that persistence pays off are all components of a positive problem-solving disposition.

Developing a positive disposition allows our students to self-monitor, check for reasonableness of their approaches and solutions and modifying their course of action, without becoming frustrated, anxious, or discouraged.



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Other Resources

[Open-Ended Questions to Promote Problem Solving](#)

[Open Response Tasks](#)

[Additional Ideas for Developing the Practice SMP 1](#)

[Planning Template](#)

[SMP 1 Notes](#)

SMP 2

Reason Abstractly and Quantitatively

Understanding the Practice

- [Why quantitative and abstract reasoning](#)
- Putting it in the abstract
- Contextualize and decontextualize

How to get there

- Number Webs
- Focus on the question
- Headline Stories

Additional Ideas for Developing the Practice



Why Q&A Reasoning

Quantitative reasoning – the ability to apply math skills and concepts to solve real problems.

It's impossible to memorize how to solve every math problem, \therefore we need to *employ abstract reasoning skills*.

(equations, variables, expressions)

SMP 2 addresses the importance of

- building a strong understanding of numbers (quantities)
- ability to represent the problem using abstractions (e.g., numbers, symbols, diagrams).
- making the connections between the problem situation and abstract representation (equation).



SMP 2

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Additional Ideas for Developing the Practice



Putting it in the abstract:

- Mathematically proficient students:
 - understand problems and quantities in the given problem
 - have the ability to convert a problem into an abstract representation using numbers, symbols, equations, diagrams, or manipulatives

- understand quantities and are able to determine when certain operations (add or subtract) or approaches (count or diagram) and can construct equations or other representations to match the problems.

[Examples at SMP 2 station]



SMP 2

Reason Abstractly and Quantitatively

Understanding the Practice

- Why quantitative and abstract reasoning
- Putting it in the abstract
- Contextualize and decontextualize
[Vignettes at SMP 2 station]

How to get there

- Number Webs
- Focus on the question
- Headline Stories

Additional Ideas for Developing the Practice

[\[Handouts at SMP 2 station\]](#)

SMP 3

Construct Viable Arguments and Critique the Reasoning of Others

Understand the Standard

- Constructing arguments
- Critiquing arguments
- Refining students' skills

How do we get there?

- Estimate It
- Agree or Disagree
- My 2 Cents

Additional Ideas for Developing the Practice

Sample Rubric



Why Construct and Critique?

As mathematicians, we construct arguments based on our mathematical thinking.

To construct arguments, we use math skills and knowledge to observe and interpret data, make conjectures about the data and situation, and draw reasonable conclusions.

We also need to listen to others' arguments, our skills include analysis and our math understanding to evaluate the arguments.

Constructing and critiquing arguments are critical components of math proficiency.

SMP 3

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*[Handouts at SMP 3 station]

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Additional Ideas for Developing the Practice

*Sample Rubric

SMP 4

Model With Mathematics

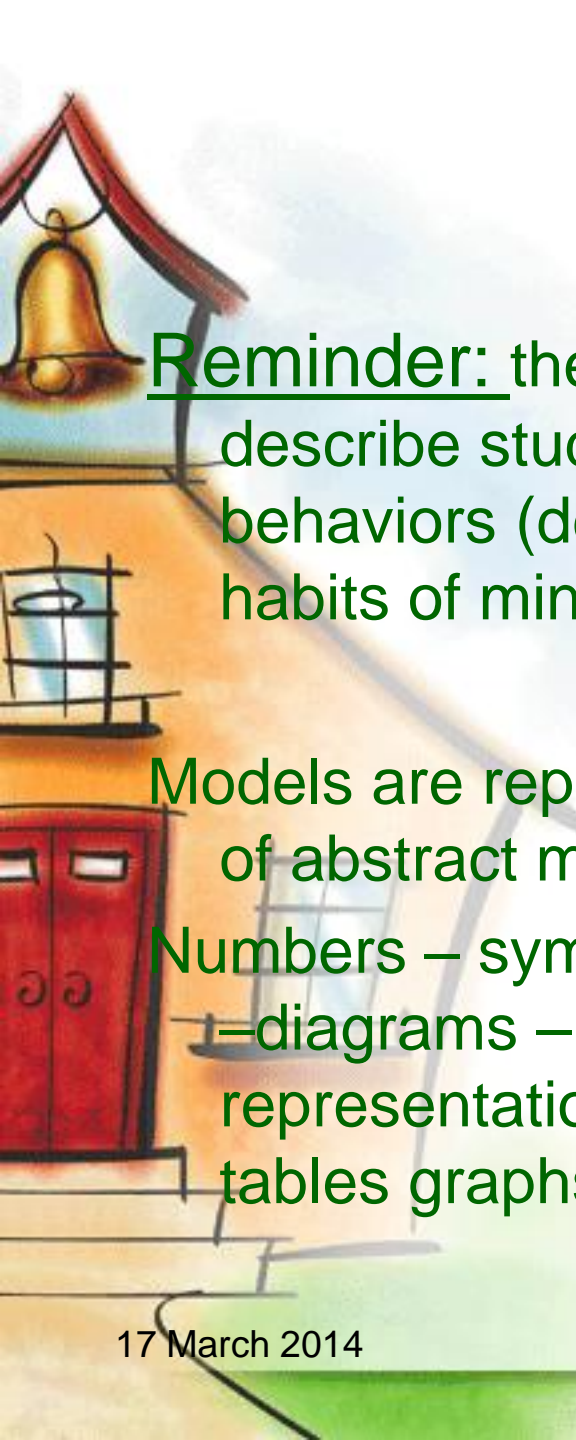
Understand the Practice

- Creating math models
- Analyzing models

How do we get there?

- Model It
- Problem-solving models
 - Part-part-whole mats for + and *
 - Bar diagrams for * and /
 - Bar diagrams for solving equations
- Technology Tools

Why Model?



Reminder: the Standards describe student behaviors (developing habits of mind)

Models are representations of abstract math data.

Numbers – symbols-objects – diagrams – graphic representations (charts, tables graphs)

Forums to manipulate math concepts and operations and to work towards solutions.

Visualize, simplify, make sense of mathematics through models.

SMP 4

Model With Mathematics

Understand the Practice

- Creating math models

- Analyzing models

[\[Handout at SMP 4 station\]](#)

How do we get there?

- Model It
- Problem-solving models
 - Part-part-whole mats for + and *
 - Bar diagrams for * and /
 - Bar diagrams for solving equations
- Technology Tools

<http://illuminations.nctm.org/>

SMP 5

Use Appropriate Tools Strategically

Understanding the Standard

- Select appropriate tools
- Use tools appropriately
 - Number lines
 - Rulers
 - Protractors
 - Graphing calculators

How do we get there?

- Number Lines as Tools
- Rulers as tools
 - The Broken Ruler
 - Exploring the magnified inch
- Developing Mental Math
 - Number Patterns
 - In my Head?



Why Focus on Tools?

Keep it mind, rather than trying to compartmentalize these Practices, think about blending the Practices to empower your students to use math and to think Mathematically.

Mathematicians use tools to do their work.

For calculations do we use paper & pencil, grab a calculator or do mental math?

For measurement – we need to determine which tool and which unit of measure and what gets measured.

Need familiarity with a variety of tools and ability to decide which ones make sense for a given task and can effectively use the tool to perform that task.

SMP 5

Use Appropriate Tools Strategically

Understanding the Standard

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How do we get there?

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- Rulers as tools
 - The Broken Ruler
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- Developing Mental Math
 - Number Patterns
 - In my Head?

[\[Handouts at SMP 5 station\]](#)



SMP 6

Attend to Precision

Understanding the Standard

- Precision in Calculations and performing math tasks

- Precision in Communication

- Communicating through words
- Communicating with Symbols

How do we get there?

- Estimate and exact
- Word walls
- Sort and Label
- Mystery words
- Translate the symbol
- Tips for writing about math
- Assessment tips



Why Focus on Precision?

We estimate in our daily lives – but there are times when math tasks must be exact.

- Paychecks
- Window blinds – need to get precise measurements and units when ordering, etc...

Math relies on precision, both in computation and in communication.

Accuracy is expected in calculations (computations) but other tasks that require precision include:

- Constructing graphs
- Measuring angles
- Determining probability of events

Communicating precisely

- using words and symbols, effectively
- describing math concepts
- explaining math procedures
- constructing math arguments



SMP 6

Attend to Precision

Understanding the Standard

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 - Communicating through words
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How do we get there?

- Estimate and exact
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[\[Handouts on SMP 6 station \]](#)

SMP 7

Look For and Make Use of Structure

Understanding the Standard

- The flexibility of numbers
- Discovering properties
- Recognizing patterns and functions

How do we get there?

- Exploring patterns and functions
 - Pattern Cover-Up
 - Patterns in the hundreds chart / multiplication chart
 - Ratio Tables to explore patterns and functions



Why Focus on Structure?

Math is quite predictable.
There is structure in math, and people who see that structure find that math makes sense.

If we understand the way math works (math properties), we know that the order in which we add (or multiply) numbers will not change the total (or product).

- Properties guide us as we explore and simplify math computations.
- Numbers are flexible – they can be broken apart and put together. (Distributive property)
- The numbers system is a system of patterns.

What patterns do you see?

$$\frac{1}{2} = 0.50$$

$$\frac{1}{3} = 0.33$$

$$\frac{1}{5} = 0.20$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{6} = 0.167$$

$$\frac{1}{10} = 0.10$$

$$\frac{1}{8} = 0.125$$

$$\frac{1}{12} = 0.083$$

$$\frac{1}{20} = 0.05$$

$$\frac{1}{16} = 0.0625$$

$$\frac{1}{24} = 0.0467$$

$$\frac{1}{40} = 0.025$$



SMP 7

Look For and Make Use of Structure

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- Discovering properties
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How do we get there?

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 - Ratio Tables to explore patterns and functions



SMP 7

Look For and Make Use of Structure

How do we get there?

Continued...

- Exploring Math Properties
 - Exploring the Commutative Property
 - Exploring the Distributive Property

- Additional Ideas for Developing the Practice

[\[Handouts for SMP 7 station\]](#)

SMP 8

Look for and Express Regularity in Repeated Reasoning

Understanding the Standard

- Exploring repetition
- Investigations to find shortcuts

How do we get there?

- Organizing and Displaying Data to Discover Rules
- Orchestrating Classroom investigations to discover shortcuts
 - Place value – adding 10 to a three-digit number
 - Formula for determining the number of combinations
 - Finding Pi



Why Focus on Repetition?

Mathematicians are observers. We observe what happens repeatedly and then figure out rules or shortcuts so we can get to answers more quickly.

We are looking for ways to minimize our efforts in mathematics through shortcuts that are the result of observations and our ability to notice and make sense of repetition.

- Patterns and properties make math predictable.
- Once students recognize and analyze what they are seeing repeatedly, they discover shortcuts – like algorithms or formulas – to make the task easier.
- Provide experiences for students to gain insights and develop the ability to understand and discover generalizations with observations of repeated events.

SMP 8

Look for and Express Regularity in Repeated Reasoning

Understanding the Standard

- Exploring repetition
- Investigations to find shortcuts

[\[Handouts for SMP 8 station\]](#)

How do we get there?

- Organizing and Displaying Data to Discover Rules
- Orchestrating Classroom investigations to discover shortcuts
 - Place value – adding 10 to a three-digit number
 - Formula for determining the number of combinations
 - Finding Pi



Work at Stations

- Review handouts
- Read through and discuss Vignettes
- Identify techniques for your grade level
- Next steps
 - back at school – discuss with grade level colleagues – identify SMPs to address and techniques for that grade level
 - Provide vertical grade level discussions – what should each grade level introduce, reinforce

Resources:

O'Connell, Susan; SanGiovanni, John.
Putting the Practices into action.
Portsmouth NH: Heinemann, 2013

Web sites:

www.heinemann.com/putting-the-practices-into-action

<http://map.mathshell.org.uk/materials/stds.php?id=1671>

https://www.mheonline.com/eminstructional/bridge2012/pdf/hoverpdf/Teaching_OR_Task.pdf



SMP 1 - Make Sense of Problems and Persevere in Solving Them

Goals	Primary K - 1 - 2	Intermediate 4 - 5 - 6	Middle Grades 6 - 7 - 8

Wrap Up

- Questions?
- What are your next steps?
- Additional materials – contact me

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